PERGAMON

International Journal of Solids and Structures 39 (2002) 251–259

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

www.elsevier.com/locate/ijsolstr

Theoretical and experimental investigations of waves in plate in magnetic field for space and averaged problems

A.G. Bagdoev, A.A. Vantsyan *

Institute of Mechanics of NAS of Armenia, Ave. Mazshal Baghzamian 24 B, 375019 Yerevan, Armenia

Received 16 March 2000

Abstract

The theoretical as well as experimental investigations of problems of vibration of magnetoelastic plates in transversal and longitudinal magnetic fields are considered.

First the space treatment is carried out. The dispersion relation for large electroconductivity in transversal field is obtained and compared with the value of the averaged treatment. The same results are obtained for longitudinal field. It is interesting that dispersion relations of space and averaged treatments for large electroconductivity do not coincide for transversal field and coincide for longitudinal field. Also the dispersion relations for bounded electroconductivity are obtained and investigated. The obtained results are compared with results of experiments.

Keywords: Plate; Magnetic field; Frequency

1. Introduction

The linear problem of vibrations of magnetoelastic bending waves in plate was considered by Ambartsumyan et al. (1977), Ambartsumyan and Bagdasaryan (1966), and Ambartsumyan and Belubekyan (1992).

Nonlinear modulation waves in longitudinal field are investigated by Bagdoev and Movsisyan (1982, 1999). All mentioned investigations are based on the classical averaged theory of thin plates. Wide class of problems of magnetoelastic waves propagation in solids is considered by Dunkin and Eringen (1963).

In the present paper the space treatment of the problem for transversal and longitudinal field is considered first for large electroconductivity and later for bounded electroconductivity. Also the averaged treatment is considered.

The two approaches are compared and it is shown that for large conductivity in longitudinal field they give the same result, but in transversal field they yield different results. For the space treatment, the longitudinal field leads to increasing of frequency, and transversal field leads to decreasing of frequency. The obtained results are checked by experiments.

* Corresponding author.

E-mail address: mechins@sci.am (A.A. Vantsyan).

0020-7683/01/$ - see front matter

PII: S0020-7683(01)00175-5

2. Space problem for transversal magnetic field

Let the undisturbed magnetic field (MF) H_0 be directed towards the z axis, normal to the plate, x, y axes are chosen in the middle plane of the plate.

Let us denote by u_x, u_y, u_z the displacement components along axes, by $\vec{H}_0 + \vec{h}$ the MF vector, where for the induced field we take

$$h_x = H_0 H'_x, \quad h_y = H_0 H'_y, \quad h_z = H_0 H'_z.$$

Although the problem is three dimensional, due to isotropy of waves properties of plate one may consider a solution in the variables x, z and set for a quasymonochromatic wave

$$\begin{aligned} u_x &= \tfrac{1}{2}U_x(z)\mathrm{e}^{\mathrm{i}\tau} + \text{c.c.}, \quad u_z = \tfrac{1}{2}U_z(z)\mathrm{e}^{\mathrm{i}\tau} + \text{c.c.}, \\ \tau &= kx - \omega t, \quad H'_x = \tfrac{1}{2}H_x(z)\mathrm{e}^{\mathrm{i}\tau} + \text{c.c.}, \quad H'_z = \tfrac{1}{2}H_z(z)\mathrm{e}^{\mathrm{i}\tau} + \text{c.c.} \end{aligned} \tag{1}$$

Because velocity of particles is much less than speed of light one can in Maxwell's equations neglect relativistic term with displacement current. As is mentioned in Ambartsumyan et al. (1977) for nonmagnetic electroconducting materials and dielectrics magnetic permeability is same.

The equations of motion and the induction in magnetoelastic media are given by Ambartsumyan et al. (1977) and by Novatski (1975) as follows:

$$\frac{b^2}{a^2}\frac{\mathrm{d}^2 U_x}{\mathrm{d}z^2} - k^2 U_x + \frac{\omega^2}{a^2}U_x + \zeta \mathrm{i}k\frac{\mathrm{d}U_z}{\mathrm{d}z} = -\frac{a_1^2}{a^2}\left(\frac{\mathrm{d}H_x}{\mathrm{d}z} - \mathrm{i}kH_z\right), \quad \zeta = 1 - \frac{b^2}{a^2}, \tag{2}$$

$$\zeta \mathrm{i}k\frac{\mathrm{d}U_x}{\mathrm{d}z} + \frac{\mathrm{d}^2 U_z}{\mathrm{d}z^2} - \frac{b^2}{a^2}k^2 U_z + \frac{\omega^2}{a^2}U_z = 0, \tag{3}$$

$$-\mathrm{i}\omega H_x + \frac{k^2}{\sigma\mu_0}H_x - \frac{1}{\sigma\mu_0}\frac{\mathrm{d}^2 H_x}{\mathrm{d}z^2} = -\mathrm{i}\omega\frac{\mathrm{d}U_x}{\mathrm{d}z}, \tag{4}$$

$$-\mathrm{i}\omega H_z + \frac{k^2}{\sigma\mu_0}H_z - \frac{1}{\sigma\mu_0}\frac{\mathrm{d}^2 H_z}{\mathrm{d}z^2} = -\omega k U_x. \tag{5}$$

Here $a_1^2 = (\mu_0 H_0^2)/\rho$, μ_0 is the magnetic permeability, σ the electroconductivity, a and b the velocities of longitudinal and transversal elastic waves, and ρ the density. Looking for the solution in the form of Novatski

$$U_z = A_j \operatorname{ch} v_j z, \quad U_x = B_j \operatorname{sh} v_j z, \quad H_x = C_j \operatorname{ch} v_j z, \quad H_z = D_j \operatorname{sh} v_j z, \tag{6}$$

where repeated indices mean summatron from Eqs. (1) to (3), one can obtain from Eqs. (2)–(6) relations between all constants and $B_{1,2,3}$ in the form

$$C_{1,2,3} = -\frac{\mathrm{i}\omega v_{1,2,3}B_{1,2,3}}{X_{1,2,3}}, \quad D_{1,2,3} = -\frac{\omega k B_{1,2,3}}{X_{1,2,3}}, \quad X_{1,2,3} = -\mathrm{i}\omega + \frac{k^2}{\sigma\mu_0} - \frac{1}{\sigma\mu_0}v_{1,2,3}^2, \tag{7}$$

$$\zeta \mathrm{i}k B_{1,2,3}v_{1,2,3} + \left(v_{1,2,3}^2 - \frac{b^2}{a^2}k^2 + \frac{\omega^2}{a^2}\right)A_{1,2,3} = 0,$$

$$\left(\frac{b^2}{a^2}v_{1,2,3}^2 - k^2 + \frac{\omega^2}{a^2}\right)B_{1,2,3} + \zeta \mathrm{i}k v_{1,2,3}A_{1,2,3} = -\frac{a_1^2}{a^2}(C_{1,2,3}v_{1,2,3} - \mathrm{i}kD_{1,2,3}).$$

From Eq. (7) one may obtain the characteristic equation for $\bar{v} = v_{1,2,3}$ in terms of ω:

$$\frac{b^2}{a^2}\bar{v}^2 - k^2 + \frac{\omega^2}{a^2} + \zeta^2\frac{k^2\bar{v}^2}{\bar{v}^2 - \frac{b^2}{a^2}k^2 + \frac{\omega^2}{a^2}} = -\frac{a_1^2}{a^2}\frac{\bar{v}^2 - k^2}{1 + \mathrm{i}\frac{k^2-\bar{v}^2}{\omega\sigma\mu_0}}. \tag{8}$$

For finite values of σ all $v_{1,2,3}^2$ are finite.
For $\sigma = \infty$ one obtains two finite values of $\bar{v}^2 = v_{1,2}^2$.
For large but finite σ third value arises $\bar{v}^2 = v_3^2$, which for $\sigma\omega\mu_0 \gg 1$:

$$\mathrm{i}\frac{v_3^2}{\omega\sigma\mu_0} = \frac{a_1^2}{b^2} + 1. \tag{9}$$

To derive the dispersion relation for the frequency ω in space treatment one must add to Eqs. (2)–(6) conditions on the plate boundaries $z = \pm(h/2)$ with dielectrics $\sigma_x = \sigma_{xz} = 0$ and conditions of continuity of $\vec{h}$.

The components of induced MF in dielectrics are

$$\bar{h}_x = \tfrac{1}{2}(C_1' \mathrm{e}^\theta + \mathrm{c.c.}), \qquad \bar{h}_z = \tfrac{1}{2}(C_2' \mathrm{e}^\theta + \mathrm{c.c.}), \quad \theta = \mathrm{i}\tau \mp kz. \tag{10}$$

Using the equation $(\partial\bar{h}_x/\partial x) + (\partial\bar{h}_z/\partial z) = 0$, one can rewrite the mentioned boundary conditions in the form

$$\begin{aligned} &C_j \operatorname{ch} v_j\frac{h}{2} = -kC_j v_j^{-1}\operatorname{sh} v_j\frac{h}{2}, \\ &B_j v_j \operatorname{ch} v_j\frac{h}{2} + \mathrm{i}kA_j \operatorname{ch} v_j\frac{h}{2} = 0, \\ &A_j v_j \operatorname{sh} v_j\frac{h}{2} + \frac{a^2 - 2b^2}{a^2}\mathrm{i}kB_j \operatorname{sh} v_j\frac{h}{2} = 0, \end{aligned} \tag{11}$$

where again summation is carried out over $j = 1, 2, 3$.

In Eq. (11) one must substitute Eq. (7). The determinant equation of Eq. (11) yields

$$\begin{vmatrix} (1 + \frac{k}{v_1}\operatorname{th} v_1\frac{h}{2})/\chi_1 & (1 + \frac{k}{v_2}\operatorname{th} v_2\frac{h}{2})/\chi_2 & (1 + \frac{k}{v_3}\operatorname{th} v_3\frac{h}{2})/\chi_3 \\ 1 + \frac{\zeta}{\Delta_1}k^2 & 1 + \frac{\zeta}{\Delta_2}k^2 & 1 + \frac{\zeta}{\Delta_3}k^2 \\ (\operatorname{th} v_1\frac{h}{2})\Gamma_1/v_1 & (\operatorname{th} v_2\frac{h}{2})\Gamma_2/v_2 & (\operatorname{th} v_3\frac{h}{2})\Gamma_3/v_3 \end{vmatrix} = 0, \tag{12}$$

where

$$\Delta_j = v_j^2 - \frac{b^2}{a^2}k^2 + \frac{\omega^2}{a^2}, \qquad \chi_j = 1 + \mathrm{i}\frac{k^2 - v_j^2}{\omega\sigma\mu_0}, \qquad \Gamma_j = \frac{a^2 - 2b^2}{a^2} - \frac{\zeta v_j^2}{\Delta_j}. \tag{13}$$

In Eq. (12) one must substitute for v_j from Eqs. (8) and (9). For finite $(\sigma\omega\mu_0)/k^2$ one obtains a complex system, which can be examined analytically for small a_1/b.

In the case of large values of $(\omega\sigma\mu_0)/k^2$, one obtains from Eq. (9) $\chi_3 = -a_1^2/b^2$, where a_1^2/b^2 is supposed small.

It is of interest, as for elastic bending waves treated by Novatski (1975), to obtain a dispersion equation on account of small addenda $v_{1,2}^2h^2$. To obtain an analytic solution one must suppose that a_1^2/b^2 is small, but the obtained values of $v_{1,2}^2$ are still rather complex. Besides, let us assume that $a_1^2/\omega^2 \ll 1$ for which one obtains simple relations.

Then Eq. (8) yields

$$v_1^2 = k^2 - \frac{\omega^2}{a^2} + \frac{a_1^2 k^2}{a^2} - \frac{a_1^4 k^4}{a^2 \omega^2 \zeta}, \tag{14}$$

$$v_2^2 = k^2 - \frac{\omega^2}{b^2} - \frac{a_1^2 k^2}{b^2} + \frac{a_1^2 \omega^2}{b^4} + \frac{a_1^4 k^4}{a^2 \omega^2 \zeta}, \tag{15}$$

where it is supposed that σ is large.

To obtain of imaginary part in $\omega = \omega_1^0 + i\omega_2^0$ for large but finite σ one can in Eq. (12) retain terms of main order. On account that in main order $v_{1,2}$ can be taken from Eqs. (14) and (15), and that thirty column in Eq. (12) for large σ yields

$$-\frac{1+\frac{k}{v_3}}{\chi a_1^2} b^2, \quad 1 - \frac{\zeta k^2}{\theta}, \quad \frac{-\frac{b^2}{a^2}}{v_3}, \quad \theta = i\omega\sigma\mu_0, \quad \chi = -\frac{\zeta k^2 - \theta}{\theta},$$

for small a_1, $v_1 h$ expanding Eq. (12) on powers of a_1, ω, kh one can show that addenda a_1^4/ω^2 do not contribute terms to Eq. (12), terms of order ω^2, $a_1^2 r$ are cancelled and the dispersion relation yields

$$-\frac{b^2}{a_1^2 \chi}\left(1 + \frac{k}{v_3} \operatorname{th} v_3 \frac{h}{2}\right)\left(\frac{\operatorname{th} v_1 \frac{h}{2}}{\operatorname{th} v_3 \frac{h}{2}} - N \frac{v_1}{v_2} \frac{1 + \frac{\zeta}{\Delta_1} k^2}{1 + \frac{\zeta}{\Delta_2} k^2}\right) + \left(1 - \frac{\zeta k^2}{\theta - \zeta k^2}\right)\left(\frac{v_1}{v_2} N - \frac{\operatorname{th} v_1 \frac{h}{2}}{\operatorname{th} v_3 \frac{h}{2}}\right)\frac{1}{1 + \frac{\zeta}{\Delta_2} k^2}$$
$$+ \frac{b^2}{a^2 v_3 \Gamma_1}\left(\frac{v_1}{\operatorname{th} v_2 \frac{h}{2}} - \frac{1 + \frac{\zeta}{\Delta_1} k^2}{1 + \frac{\zeta}{\Delta_2} k^2} \frac{v_1}{\operatorname{th} v_2 \frac{h}{2}}\right) = 0, \quad N = \frac{\Gamma_2}{\Gamma_1}. \tag{16}$$

Retaining small terms of main order, on account that $v_3 \approx \sqrt{-i\omega\mu_0\sigma}$, $\chi \approx 1$, one obtains for small dissipation

$$\omega^2 = \frac{h^2}{3} b^2 k^4 \zeta - \frac{2a_1^2 k^2 b^2}{a^2 \zeta} - a_1^2 \frac{2}{h} \frac{k^2}{v_3}. \tag{17}$$

Then one can obtain in main order

$$(\omega_1^0)^2 = \frac{h^2}{3} b^2 k^4 \zeta - \frac{2a_1^2 b^2 k^2}{a^2 \zeta}, \tag{18}$$

$$\omega_2^0 = -\frac{a_1^2 k^2}{h\omega_1^0 \sqrt{2\sigma\mu_0\omega_1^0}}. \tag{19}$$

One can also obtain a simple dispersion relation for bounded σ and small a_1 in form

$$\omega^2 = \frac{h^2}{3} b^2 k^4 \zeta - a_1^2 k^2 \frac{1 + \frac{b^2}{a^2}}{\zeta}. \tag{20}$$

3. Dispersion relation in averaged treatment

For comparison with averaged treatment for arbitrary values of a_1/b one can use the averaged equations of bending of plates.

For the displacements the classical theory yields

$$u_z = u(x,t), \qquad u_x = -z\frac{\partial u}{\partial x}, \qquad u = \tfrac{1}{2}(Ae^{i\tau} + \text{c.c.}). \tag{21}$$

The equations of motion and induction by Ambartsumyan et al. (1977) yield

$$\begin{aligned} &D\frac{\partial^4 u}{\partial x^4} + \rho h\frac{\partial^2 u}{\partial t^2} = Z, \\ &\frac{\partial h_x}{\partial t} = -H_0\frac{\partial v_z}{\partial x} + \frac{1}{\sigma\mu_0}\left(\frac{\partial^2 h_x}{\partial x^2} + \frac{\partial^2 h_x}{\partial z^2}\right), \\ &\frac{\partial h_z}{\partial t} = -H_0\frac{\partial v_x}{\partial x} + \frac{1}{\sigma\mu_0}\left(\frac{\partial^2 h_z}{\partial x^2} + \frac{\partial^2 h_z}{\partial z^2}\right), \end{aligned} \tag{22}$$

where

$$v_{x,z} = \frac{\partial u_{x,z}}{\partial t}, \quad D = \frac{Eh^3}{12(1-\nu^2)}, \tag{23}$$

$$Z = \rho\int_{-h/2}^{h/2}\left(K_z + z\frac{\partial K_x}{\partial x}\right)dz, \quad K_z = 0, \;\; K_x = \frac{1}{\rho}\mu_0 H_0\left(\frac{\partial h_x}{\partial z} - \frac{\partial h_z}{\partial x}\right). \tag{24}$$

E, ν are Young's modulus and Poisson coefficient. Matching the solution of Eqs. (22) and (24) with the MF in dielectrics $\tilde{h}_x$, $\tilde{h}_z$ one can obtain the dispersion relation for arbitrary σ

$$Dk^4 - \rho h\omega^2 = -i\mu_0 H_0^2 k^2\left(\frac{k^3h^3}{12} + 2\frac{\lambda_1^2 - k^2}{\lambda_1^3}\,\frac{\lambda_1\frac{h}{2}\operatorname{ch}\lambda_1\frac{h}{2} - \operatorname{sh}\lambda_1\frac{h}{2}}{\operatorname{ch}\lambda_1\frac{h}{2} + \frac{k}{\lambda_1}\operatorname{sh}\lambda_1\frac{h}{2}}\right)\frac{\omega}{i\omega - \frac{k^2}{\sigma\mu_0}}, \tag{25}$$

where $\lambda_1 = (k^2 - i\omega\sigma\mu_0)^{1/2}$.

Eq. (25) coincides with the equation obtained by another method by Bagdasaryan and given by Ambartsumyan and Belubekyan (1992).

For bounded σ one assumes $\lambda_1 h \ll 1$ in Eq. (25):

$$Dk^4 - \rho h\omega^2 = \frac{1}{12}\rho a_1^2 k^2 h^3 i\omega\sigma\mu_0,$$

which coincides with the equation by Ambartsumyan and Bagdasaryan (1966).

The obtained relation does not coincide with Eq. (20). For large σ and λ_1 one obtains from Eq. (25)

$$(\omega_1^0)^2 = \frac{1}{\rho h}(Dk^4 + \mu_0 H_0^2 k^2 h), \qquad \omega_2^0 = -\frac{\mu_0 H_0^2 k^2}{\sqrt{2\sigma\mu_0}\rho h(\omega_1^2)^{3/2}}. \tag{26}$$

The second relation in Eq. (26) coincides with Eq. (19), but the first one is different from the space solution Eq. (18), leading to increasing of $(\omega_1^0)^2$ due to H_0, whereas Eq. (18) leads to decreasing of $(\omega_1^0)^2$ due to H_0. Thus for transversal MF there is quantitative and qualitative difference for dispersion relations obtained by space and averaged treatment. So Kircehhoff hypotheses for magnetoelastic plate is not applicable.

4. Case of longitudinal magnetic field

Let the undisturbed MF $\overline{H}_0$ be directed along the x axis, then one can write instead of Eqs. (2)–(5)

$$\begin{aligned}
&\frac{b^2}{a^2}\frac{\mathrm{d}^2U_x}{\mathrm{d}z^2} - k^2U_x + \frac{\omega^2}{a^2}U_x + \zeta\mathrm{i}k\frac{\mathrm{d}U_z}{\mathrm{d}z} = 0,\\
&\zeta\mathrm{i}k\frac{\mathrm{d}U_x}{\mathrm{d}z} + \frac{\mathrm{d}^2U_z}{\mathrm{d}z^2} - \frac{b^2}{a^2}k^2U_z + \frac{\omega^2}{a^2}U_z = \frac{a_1^2}{a^2}\left(\frac{\mathrm{d}H_x}{\mathrm{d}z} - \mathrm{i}kH_z\right),\\
&-\mathrm{i}\omega H_x + \frac{k^2}{\sigma\mu_0}H_x - \frac{1}{\sigma\mu_0}\frac{\mathrm{d}^2H_x}{\mathrm{d}z^2} = \mathrm{i}\omega\frac{\mathrm{d}U_z}{\mathrm{d}z},\\
&-\mathrm{i}\omega H_z + \frac{k^2}{\sigma\mu_0}H_z - \frac{1}{\sigma\mu_0}\frac{\mathrm{d}^2H_z}{\mathrm{d}z^2} = \omega kU_z,
\end{aligned} \tag{27}$$

U_z, U_x are given by Eq. (6) but

$$H_x = C_j\,\mathrm{sh}\,v_jz, \quad H_z = D_j\,\mathrm{ch}\,v_jz \tag{28}$$

where summation is carried out over j from Eqs. (1) to (3).

Substituting Eq. (28) in Eq. (27) one obtains

$$\bar{v}^2 - \frac{b^2}{a^2}k^2 + \frac{\omega^2}{a^2} + \frac{\zeta^2\bar{v}^2k^2}{\frac{b^2}{a^2}\bar{v}^2 - k^2 + \frac{\omega^2}{a^2}} = -\frac{a_1^2}{a^2}\frac{\bar{v}^2 - k^2}{1 + \mathrm{i}\frac{k^2-\bar{v}^2}{\omega\sigma\mu_0}}. \tag{29}$$

Repeating the procedure of Section 2, satisfying boundary conditions on the plate edges one obtains the dispersion relation for large σ

$$\frac{\mathrm{th}\,v_1\frac{h}{2}}{\mathrm{th}\,v_2\frac{h}{2}} - \frac{v_1}{v_2}\frac{1+\zeta\frac{v_1^2}{\Delta_1'}}{1+\zeta\frac{v_2^2}{\Delta_2'}}N' - \frac{a_1^2}{a^2}\frac{2k}{h}\left(1-\frac{k}{v_3}\right)\frac{v_1}{v_2}\frac{1}{1+\zeta\frac{v_2^2}{\Delta_2'}}\frac{\frac{\zeta v_2^2}{\Delta_2'} - \frac{\zeta v_1^2}{\Delta_1'}}{\Gamma_1'} = 0, \quad N' = \frac{\Gamma_2'}{\Gamma_1'}, \tag{30}$$

where

$$\Delta_{1,2}' = \frac{b^2}{a^2}v_{1,2}^2 - k^2 + \frac{\omega^2}{a^2}, \quad \Gamma_{1,2}' = \frac{a^2-2b^2}{a^2}\frac{\zeta v_{1,2}^2k^2}{\Delta_{1,2}'} + v_{1,2}^2,$$

$$1 - \mathrm{i}\frac{v_3^2}{\omega\sigma\mu_0} = -\frac{a_1^2}{a^2}.$$

The roots of Eq. (29) may be obtained similarly to Eqs. (14) and (15), and for finite θ yield

$$v_1^2 = k^2 - \frac{\omega^2}{a^2} - \frac{a_1^2}{a^2}k^2 + \frac{\omega^2a_1^2}{a^4} - \frac{b^2a_1^4k^4}{\zeta\omega^2a^4} - \frac{a_1^2\omega^2k^2}{a^4\theta}, \tag{31}$$

$$v_2^2 = k^2 - \frac{\omega^2}{b^2} + \frac{a_1^2}{b^2}k^2 + \frac{a_1^4k^4}{b^2\zeta^2\omega^2} - \frac{a_1^4k^4}{b^2\omega^2} + \frac{a_1^2\omega^2k^2}{b^4\theta}. \tag{32}$$

One can show that addenda a_1^4/ω^2 do not contribute to the dispersion relation, and for large σ from Eq. (30) one obtains

$$(\omega_1^0)^2 = \frac{h^2b^2}{3}k^4\zeta + \frac{2a_1^2k}{h}, \tag{33}$$

$$\omega_2^0 = -\frac{k^2 a_1^2}{h(\omega_1^0)^{3/2}\sqrt{2\sigma\mu_0}}. \tag{34}$$

This investigation was carried out by Bagdoev and Sahakyan (1999).

For comparison with the averaged treatment one repeats calculations carried out for the transversal field and obtains in the averaged treatment the following dispersion relation for the longitudinal field:

$$Dk^4 - \rho h\omega^2 = \frac{H_0^2\mu_0 k\omega}{-\mathrm{i}\omega + \frac{1}{\sigma\mu_0}k^2}\left(\mathrm{i}kh - 2\mathrm{i}\frac{k^2-\lambda_1^2}{\mathrm{sh}\,\lambda_1\frac{h}{2} + \frac{k}{\lambda_1}\mathrm{ch}\,\lambda_1\frac{h}{2}}\frac{\mathrm{sh}\,\lambda_1\frac{h}{2}}{\lambda_1^2}\right). \tag{35}$$

Hence for large σ and λ_1 one obtains a dispersion relation which, in contrast to the case of the transversal field, coincides with the relation of space problems (33) and (34).

Frequency (33) coincides and attenuation (34) does not coincide with the values obtained from particular considerations by Bagdoev and Movsisyan (1999).

For the bounded σ, $\lambda_1 h \ll 1$ and Eq. (35) yields in the averaged treatment

$$Dk^4 - \rho h\omega^2 = \mathrm{i}\rho a_1^2\sigma\mu_0\omega h.$$

One can also obtain from Eqs. (27) and (28) the dispersion relation for bounded values of θ in space treatment after lengthy calculations in the form

$$(\omega_1^0)^2 = \frac{h^2b^2k^4}{3}\zeta + a_1^2k^2\left(1 + \frac{2b^4}{a^4\zeta} + \frac{2b^2}{a^2}\right), \qquad \omega_2^0 = -\frac{a_1^2\sigma\mu_0}{2}. \tag{36}$$

It is seen from Eq. (36) and from the above mentioned averaged dispersion relation that the second equation (36) coincides with the averaged value of ω_2^0 but $(\omega_1^0)^2$ is quite different.

5. Experimental investigation of vibration of plates in transversal and longitudinal fields

The investigation of influence of constant MF on frequencies of free oscillations of plate are carried out by scheme given on Fig. 1.

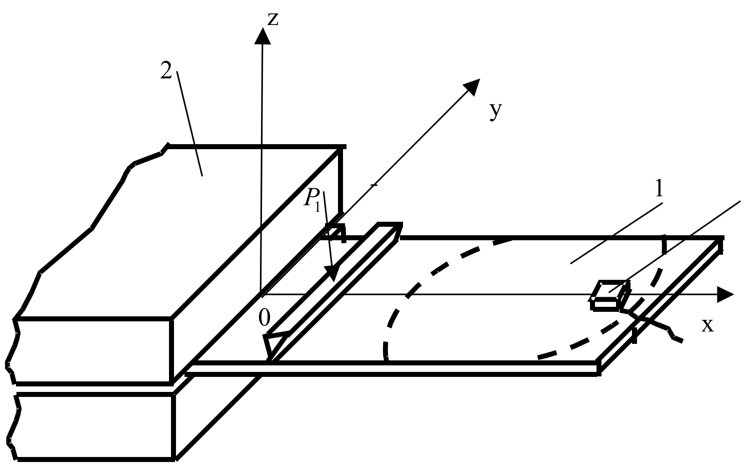

Fig. 1. The principle scheme of experiment.

Plate 1 is rigidly fastened by holder 2 and is located in longitudinal MF, directed along Ox axis or in transversal MF directed along Oz axis. Longitudinal MF $H_{01} = H_0$ is generated by means of solenoid whose axis is directed along Ox.

In order to study the influence of transversal MF $H_{03} = H_0$ the plate is located between poles of other magnet. In contrast to the case of longitudinal field which was applied to all surface of plate, the transversal field was applied within the region mentioned in the figure by dotted lines.

For the determination of free frequencies of the plate, on small distance from rigidly fastened end by force $P_1 = P_0 \sin \omega t$, produced by smoothly regulated on frequency and amplitude vibrator, are generated vibrations.

By means of light sensor 3, which was fastened rigidly with plate, signals from vibrating plate were given to entrance of oscillograph C8-17.

In moment of abrupt increasing of the amplitude of vibration of the plate which corresponds to resonance, the frequency of vibration was fixed. Then MF (H_{01} or H_{03}) was produced for which other resonance frequency was obtained.

It was observed any increasing of frequency for longitudinal field and any decreasing of frequency for transversal field. The value of the MF was $H_{03} = 0.15$–0.2 T and of longitudinal field $H_{01} \sim 10^{-2}$ T.

For the mentioned MF, the increase of frequency of vibration in longitudinal field was about 1–2%. Decreasing of frequency of free vibrations for the transversal field was ~2–3%.

Experiments were carried out for aluminum plates of sizes $360 \times 60 \times 1$, $360 \times 60 \times 4$, $360 \times 60 \times 5$ mm^3 and for cuprum plate of size $310 \times 70 \times 1$ mm^3.

Measurements of MF were carried out by nuclear-magnetic resonance method using device ϕ1-1. Experimental data are given in the following table:

$2h$ (mm)	H_{01}		H_{03}	
	ω_{00}	ω_H	ω_{00}	ω_H
1	44	44.5	44	42.5
4	42	42.6	42	40.5
5	41	41.8	41	39.6
1	32.8	33.6	32.8	31.2

where $2h$ is the thickness of plate, first three lines correspond to aluminum and fourth to cuprum, ω_{00} is the frequency of free vibrations without MF, and ω_H is the real part of frequency in presence of MF.

The second and third line in the table correspond to first harmonics and first line to second harmonics.

As is seen from the table, values of ω_{00}, ω_H are in qualitative and quantitative agreement with values from Eqs. (18) and (33) which show decreasing of frequency for transversal fields and increasing for longitudinal fields. It must be noted that for the considered plates the wave number $k = (\pi/360)$ mm$^{-1} \approx 10^{-2}$ mm^{-1} and for the dimensionless parameter $\sigma\omega H_0/k^2$ large values are obtained which corresponds to the considered theory.

6. Concluding remarks

In the present paper is given scrupulous investigation of dispersion relations for bending magnetoelastic waves in transversal and longitudinal field.

It is shown that for transversal field more exact space treatment gives decreasing of frequency due to presence of MF and averaged treatment based on Kircehhoff hypothesis for plates gives increasing of frequency i.e. incorrect results. In case of longitudinal MF both treatments give the same result. Mentioned theories and their results are justified by means of experiments.

References

Ambartsumyan, S.A., Bagdasaryan, G.E., Belubekyan, M.V., 1977. Magnetoelasticity of Thin Shells and Plates. Nauka, Moskow, 272p (in Russian).

Ambartsumyan, S.A., Bagdasaryan, G.E., 1966. Electroconducting plates and shells in the magnetic field. Phys.-Math. Literature, 287p (in Russian).

Ambartsumyan, S.A., Belubekyan, M.V., 1992. Vibrations and Stability of the Current-carrying Elastic Plates. Edition of NAS Armenia, Yerevan, 124p (in Russian).

Bagdoev, A.G., Movsisyan, L.A., 1982. Nonlinear vibrations of plates in longitudinal magnetic field. Izvestia AN Arm. SSR Mech. 35 (1), 16–22 (in Russian).

Bagdoev, A.G., Movsisyan, L.A., 1999. Thermomagnetoelastic modulation waves in non-linear plate. Izvestia NAS Arm. Mech. 52 (1), 25–29.

Bagdoev, A.G., Sahakyan, S.G., 1999. The stability of nonlinear modulation waves in plate in magnetic field for space and averaged problems. In: Informative Technologies and Management. Noyan Tapan, Yerevan, pp. 95–101.

Dunkin, J.W., Eringen, A.C., 1963. On the propagation of waves in an electromagnetic elastic solid. Int. J. Engng. Sci. 1, 461–495.

Novatski, V., 1975. Theory of Elasticity. Mir, Moscow, 863p (in Russian).